

Holographic Superconductors in $z = 3$ Hořava-Lifshitz gravity without condition of detailed balance

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Abstract

We study holographic superconductors in a Hořava-Lifshitz black hole without the condition of the detailed balance. We show that it is easier for the scalar hair to form as the parameter of the detailed balance becomes larger, but harder when the mass of the scalar field larger. We also find that the ratio of the gap frequency in conductivity to the critical temperature, ω_g/T_c , almost linear decreases with the increase of the balance constant. For $\epsilon = 0$ the ratio reduces to Cai's result $\omega_g/T_c \approx 13$ found in the Hořava-Lifshitz black hole with the condition of the detailed balance, while as $\epsilon \rightarrow 1$ it tends to Horowitz-Roberts relation $\omega_g/T_c \approx 8$ obtained in the AdS Schwarzschild black hole. Our result provides a bridge between the results for the Hořava-Lifshitz theory with the condition of the detailed balance and Einstein's gravity.

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I. INTRODUCTION

The AdS/CFT correspondence [1–3] relates a weak coupling gravity theory in an anti-de Sitter space to a strong coupling conformal field theory in one less dimensions. Recently it has been applied to condensed matter physics and in particular to superconductivity [4, 5]. In the pioneering papers Gubser [4, 5] suggested that near the horizon of a charged black hole there is in operation a geometrical mechanism parametrized by a charged scalar field of breaking a local $U(1)$ gauge symmetry. Then, the gravitational dual of the transition from normal to superconducting states in the boundary theory was constructed. This dual consists of a system with a black hole and a charged scalar field, in which the black hole admits scalar hair at temperature lower than a critical temperature, but does not possess scalar hair at higher temperatures [6]. In this system a scalar condensate can take place through the coupling of the scalar field with the Maxwell field of the background. Much attention has been focused on the application of AdS/CFT correspondence to condensed matter physics since then [7–19].

Höřava [20, 21] proposed a new class of quantum gravity. The key property of this theory is the three dimensional general covariance and time re-parameterization invariance. It is this anisotropic rescaling that makes Höřava’s theory power-counting renormalizable. Therefore, many authors pay their attention to this gravity theory and its cosmological and astrophysical applications, and found many interesting results [22–35]. These investigations imply that there exists the distinct difference between the Höřava-Lifshitz theory and Einstein’s gravity.

Recently, in order to see what difference will appear for the holographic superconductivity in the Höřava-Lifshitz theory, compared with the case of the relativistic general relativity, Cai *et al.* [36] studied the phase transition of planar black holes in the Höřava-Lifshitz gravity with the condition of the detailed balance in which the metric function is described by $f(r) = x^2 - \sqrt{c_0 x}$. They argued that the holographic superconductivity is a robust phenomenon associated with asymptotic AdS black holes. And they also got a relation connecting the gap frequency in conductivity with the critical temperature, which is given by $\frac{\omega_g}{T_c} \approx 13$, with the accuracy more than 93% for a range of scalar masses.

Note that the Höřava-Lifshitz black hole without the condition of the detailed balance has rich physics [37–39], i.e., changing the parameter of the detailed balance ϵ from 0 to 1 it can produce the different black holes for the Höřava-Lifshitz theory and Einstein’s gravity. Thus, it seems to be an interesting topic to consider the effects of the parameter of the detailed balance on the scalar conden-

sation formation, the electrical conductivity, and the ratio ω_g/T_c which connects the gap frequency in conductivity with the critical temperature.

The paper is organized as follows. In Sec. 2 we present black holes with hyperbolic horizons in Hořava-Lifshitz gravity in which the action without the condition of the detailed balance. In Sec. 3 we explore the scalar condensation in the Hořava-Lifshitz black hole background by numerical and analytical approaches. In Sec. 4 we study the electrical conductivity and find ratio of the gap frequency in conductivity to the critical temperature. We summarize and discuss our conclusions in the last section.

II. BLACK HOLE WITH HYPERBOLIC HORIZON IN $z = 3$ HOŘAVA-LIFSHITZ GRAVITY

In non-relativistic field theory, space and time have different scalings, which is called anisotropic scaling, $x^i \rightarrow bx^i$, $t \rightarrow b^z t$, $i = 1, 2, 3$, where z is called *dynamical critical exponent*. In order for a theory to be power counting renormalizable, the critical exponent has at least $z = 3$ in four spacetime dimensions. For $z = 3$, the action without the condition of the detailed balance for the Hořava-Lifshitz theory can be expressed as [37, 38]

$$I = \int dt d^3x [\mathcal{L}_0 + (1 - \epsilon^2)\mathcal{L}_1], \quad (2.1)$$

with

$$\begin{aligned} \mathcal{L}_0 &= \sqrt{g}N \left\{ \frac{2}{\kappa^2}(K_{ij}K^{ij} - \lambda K^2) + \frac{\kappa^2\mu^2(\Lambda R - 3\Lambda^2)}{8(1 - 3\lambda)} \right\}, \\ \mathcal{L}_1 &= \sqrt{g}N \left\{ \frac{\kappa^2\mu^2(1 - 4\lambda)}{32(1 - 3\lambda)}R^2 - \frac{\kappa^2}{2\omega^4} \left(C_{ij} - \frac{\mu\omega^2}{2}R_{ij} \right) \left(C^{ij} - \frac{\mu\omega^2}{2}R^{ij} \right) \right\}, \\ K_{ij} &= \frac{1}{2N}(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i), \\ C^{ij} &= \epsilon^{ikl} \nabla_k \left(R^j_l - \frac{1}{4}R\delta^j_l \right) = \epsilon^{ikl} \nabla_k R^j_l - \frac{1}{4}\epsilon^{ikj} \partial_k R, \end{aligned}$$

where κ^2 , μ , Λ , and ω are constant parameters, ϵ is parameter of the detailed balance ($0 < \epsilon \leq 1$), N^i is the shift vector, K_{ij} is the extrinsic curvature and C_{ij} the Cotten tensor. It is interesting to note that the action (2.1) reduces to the action in Ref. [38] if $\epsilon = 0$, and it becomes the action for the Einstein's gravity if $\epsilon = 1$.

From the action (2.1), Cai *et al.* [39] found a static black hole with hyperbolic horizon whose horizon has an arbitrary constant scalar curvature $2k$ with $\lambda = 1$. The line element of the black hole

can be expressed as

$$ds^2 = -N^2(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_k^2, \quad (2.2)$$

with

$$N^2 = f = k + \frac{x^2}{1 - \epsilon^2} - \frac{\sqrt{\epsilon^2 x^4 + (1 - \epsilon^2)c_0 x}}{1 - \epsilon^2}, \quad (2.3)$$

where $x = \sqrt{-\Lambda} r$, $k = -1, 0, 1$, and $c_0 = (x_+^4 + 2kx_+ + (1 - \epsilon^2)k^2)/x_+$ in which x_+ is the horizon radius of the black hole, i.e., the largest root of $f(r) = 0$. Comparing with the standard AdS_4 spacetime, we may set $\frac{-\Lambda}{1+\epsilon} = \frac{1}{L_{\text{AdS}}^2}$, where L_{AdS} is the radius of AdS_4 . The authors in ref. [39] also found that the solutions has a finite mass $M = \kappa^2 \mu^2 \Omega_k \sqrt{-\Lambda} c_0 / 16$. For $\epsilon = 0$, the solution goes back to the solution in Ref. [38]. Furthermore, when $\epsilon = 1$, the solution becomes the (A)dS Schwarzschild black hole.

The Hawking temperature of the black hole is

$$T = \frac{\sqrt{-\Lambda}}{8\pi} \frac{3x_+^4 + 2kx_+^2 - (1 - \epsilon^2)k^2}{x_+(x_+^2 + (1 - \epsilon^2)k)}, \quad (2.4)$$

which is always a monotonically increasing function of horizon radius x_+ in the physical regime. This implies that the black holes with hyperbolic horizons in the Hořava-Lifshitz theory are thermodynamically stable.

III. SCALAR CONDENSATION IN HOŘAVA-LIFSHITZ BLACK-HOLE BACKGROUND

Now, we study the scalar condensation in the Hořava-Lifshitz gravity. In the background of the black hole described by Eq. (2.3) with $k = 0$, i.e.,

$$N^2 = f = \frac{x^2}{1 - \epsilon^2} - \frac{\sqrt{\epsilon^2 x^4 + (1 - \epsilon^2)xx_+^3}}{1 - \epsilon^2}, \quad (3.1)$$

we consider a Maxwell field and a charged complex scalar field with the action

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - |\nabla\psi - iA\psi|^2 - V(|\psi|) \right], \quad (3.2)$$

where $F^{\mu\nu}$ is the Maxwell field strength $F = dA$ and ψ is the complex scalar field with the potential $V = m^2|\psi|^2$. We focus our attention on the case that these fields are weakly coupled to gravity, i.e., they do not backreact on the metric of the spacetime. Thus, we can take the ansatz

$$\begin{aligned} A_\mu &= (\phi(r), 0, 0, 0), \\ \psi &= \psi(r). \end{aligned} \quad (3.3)$$

This ansatz implies that the phase factor of the complex scalar field is a constant. Therefore, we may take ψ to be real. In the background of the black hole described by Eqs. (2.2) and (3.1), the equations of the scalar field $\psi(r)$ and the scalar potential $\phi(r)$ are given by

$$\psi'' + \left(\frac{f'}{f} + \frac{2}{r} \right) \psi' + \left(\frac{\phi^2}{f^2} - \frac{m^2}{f} \right) \psi = 0, \quad (3.4)$$

$$\phi'' + \frac{2}{r} \phi' - \frac{2\psi^2}{f} \phi = 0, \quad (3.5)$$

where a prime denotes the derivative with respect to r .

At the event horizon $r = r_+$, we must have

$$\begin{aligned} \psi(r_+) &= -\frac{3\psi'(r_+)}{2m^2 L^2}, \\ \phi(r_+) &= 0, \end{aligned} \quad (3.6)$$

because their norms are required to be finite, where $L^2 = L_{AdS}^2/(1 + \epsilon)$. And at the asymptotic region ($r \rightarrow \infty$), the solutions behave like

$$\begin{aligned} \psi &= \frac{\psi_-}{r^{\lambda_-}} + \frac{\psi_+}{r^{\lambda_+}}, \\ \phi &= \mu - \frac{\rho}{r}, \end{aligned} \quad (3.7)$$

with

$$\lambda_{\pm} = \frac{1}{2} \left[3 \pm \sqrt{9 + 4m^2 L_{AdS}^2} \right], \quad (3.8)$$

where μ and ρ are interpreted as the chemical potential and charge density in the dual field theory, respectively. Because the boundary is a (2+1)-dimensional field theory, μ is of mass dimension one and ρ is of mass dimension two. We can read off the expectation values of operator \mathcal{O} dual to the field ψ . From Ref. [40], we know that for ψ , both of these falloffs are normalizable, and in order to keep the theory stable, we should either impose

$$\psi_- = 0, \quad \text{and} \quad \langle \mathcal{O}_+ \rangle = \psi_+, \quad (3.9)$$

or

$$\psi_+ = 0, \quad \text{and} \quad \langle \mathcal{O}_- \rangle = \psi_-. \quad (3.10)$$

Note that the dimension of temperature T is of mass dimension one, the ratio T^2/ρ is dimensionless. Therefore increasing ρ while T is fixed, is equivalent to decrease T while ρ is fixed. In our calculation,

we find that when $\rho > \rho_c$, the operator condensate will appear; this means when $T < T_c$ there will be an operator condensate, that is to say, the superconducting phase occurs.

In what following we will present a detailed analysis of the condensation of the operator O_+ by numerical and analytical methods, respectively.

A. Numerical analysis of condensation

The equations (3.4) and (3.5) can be solved numerically by doing integration from the horizon out to the infinity with the boundary conditions mentioned above.

Changing the value of ϵ , we present in Fig. 1 the influence of the parameter of the balance on the condensation with fixed values $m^2 L_{AdS}^2 = 0, -1$ and -2 , and in Fig. 2 the critical temperature as a function of the balance parameter with fixed values $m^2 L_{AdS}^2 = 0, -1$ and -2 . We know from the figures that as the parameter of the detailed balance increases with fixed mass of the scalar field, the condensation gap becomes smaller, corresponding to larger the critical temperature, which means that the scalar hair can be formed easier for the larger ϵ .

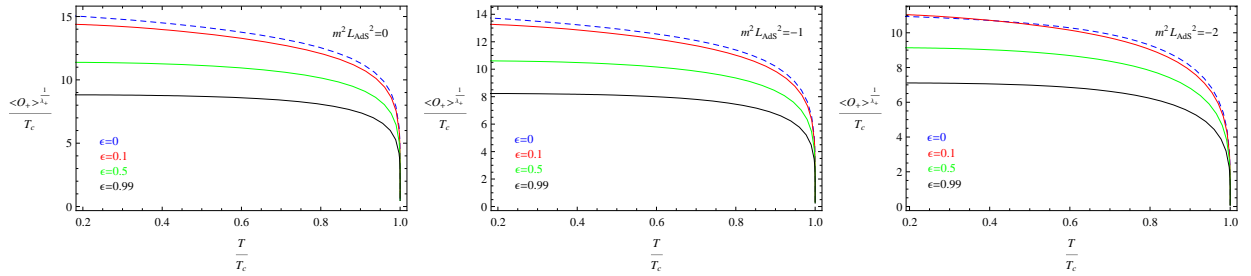


FIG. 1: (Color online) The condensate as a function of the temperature with fixed values $m^2 L_{AdS}^2 = 0, -1$ and -2 . The four lines from top to bottom correspond to increasing ϵ , i.e., 0 (blue), 0.1 (red), 0.5 (green) and 0.99 (black), respectively. It is shown that the condensation gap becomes smaller as ϵ increases for the same mass $m^2 L_{AdS}^2$.

Altering the value of $m^2 L_{AdS}^2$, we show in Fig. 3 the influence of the mass on the condensation with fixed values $\epsilon = 0$ and 0.1. It is clear that for the same ϵ , the condensation gap becomes larger if m^2 becomes less negative, which means that the scalar hair can be formed more difficult as m^2 becomes less negative.

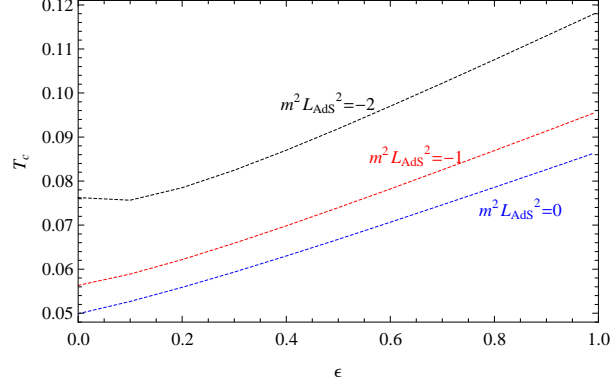


FIG. 2: (Color online) The critical temperature as a function of the balance parameter with fixed values $m^2 L_{AdS}^2$. The three lines from top to bottom correspond to $m^2 L_{AdS}^2 = -2$ (black), -1 (red) and 0 (blue), respectively.

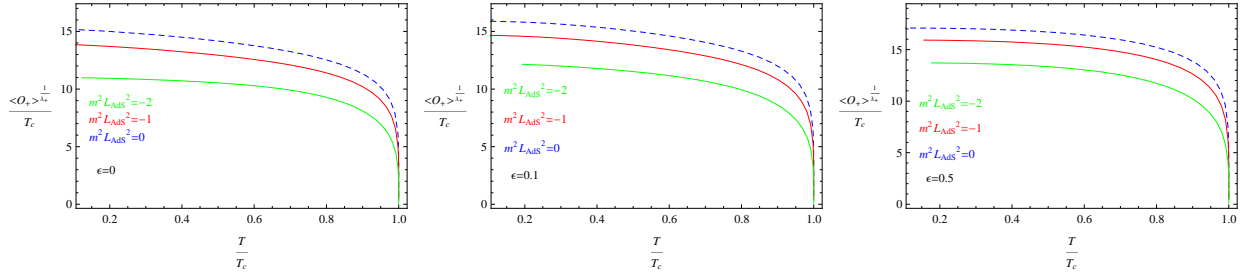


FIG. 3: (Color online) The condensate as a function of temperature with fixed values ϵ for the various masses of the scalar field. The three lines from bottom to top correspond to increasing $m^2 L_{AdS}^2$, i.e., $m^2 L_{AdS}^2 = -2$ (green), -1 (red), 0 (blue), respectively. It is shown that the condensation gap becomes larger if m^2 becomes less negative for fixed ϵ .

B. Analytical understanding of condensation

A semi-analytical method can also be applied in understanding the condensation although the Eqs. (3.4) and (3.5) are coupled and nonlinear. The method consists in finding approximate solutions near the horizon and in the asymptotic AdS space and then smoothly matching the solutions at an intermediate point [41]. In particular in Ref. [42] an analytical expression for the critical temperature was obtained and the phase transition phenomenon was demonstrated. And the result obtained in this way is in a good agreement with the numerical result. In this section we use this analytical approach to study the condensation of the scalar operator in the Hořava-Lifshitz black hole, and then compare the result with that obtained by numerical method.

Take a change $z = r_+/r$, Eqs. (3.4) and (3.5) can be rewritten as

$$\psi'' + \frac{f'}{f}\psi' + \frac{r_+^2}{z^4}\left(\frac{\phi^2}{f^2} - \frac{m^2}{f}\right)\psi = 0, \quad (3.11)$$

$$\phi'' - \frac{r_+^2}{z^4}\frac{2\psi^2}{f}\phi = 0, \quad (3.12)$$

where the prime denotes differentiation with z . Regularity of the functions at the horizon $z = 1$ requires

$$\begin{aligned} \psi(1) &= \frac{3}{2m^2L^2}\psi'(1), \\ \phi(1) &= 0. \end{aligned} \quad (3.13)$$

And near the boundary $z = 0$ we have

$$\begin{aligned} \psi &= C_- z^{\lambda_-} + C_+ z^{\lambda_+}, \\ \phi &= \mu - \frac{\rho}{r_+} z. \end{aligned} \quad (3.14)$$

We will set $C_- = 0$ and fix ρ in the following discussion. With the help of the regular horizon boundary condition (3.13), the leading order approximate solutions near the horizon, $z = 1$, for the Eqs. (3.11) and (3.12) can be expressed as

$$\begin{aligned} \psi(z) &= a \left\{ 1 + \frac{2m^2L^2}{3}(1-z) \right. \\ &\quad \left. + \frac{L^2}{36} \left[(3 + 9\epsilon^2 + 4m^2L^2)m^2 - \frac{4L^2b^2}{r_+^2} \right] (1-z)^2 + \dots \right\}, \end{aligned} \quad (3.15)$$

$$\phi(z) = b \left[(1-z) + \frac{2L^2a^2}{3}(1-z)^2 + \dots \right], \quad (3.16)$$

where $a \equiv \psi(1)$ and $b \equiv -\phi'(1)$ with $a, b > 0$ which makes $\psi(z)$ and $\phi(z)$ positive near the horizon. Matching smoothly the solutions (3.15), (3.16) with (3.14) at an intermediate point z_m with $0 < z_m < 1$, we have

$$C_+ = \frac{6 + 2m^2L^2(1 - z_m)}{3[2z_m + \lambda_+(1 - z_m)]z_m^{\lambda_+-1}} a, \quad (3.17)$$

$$b = \frac{r_+}{2L^2} \sqrt{\frac{A}{[\lambda_+ - (\lambda_+ - 2)z_m](1 - z_m)}} \equiv \frac{\tilde{b}r_+}{L^2}, \quad (3.18)$$

$$a^2 = \frac{3}{4L^2(1 - z_m)} \left(\frac{\rho}{br_+} \right) \left(1 - \frac{br_+}{\rho} \right), \quad (3.19)$$

with

$$\begin{aligned} A &= 4L^4m^4(1 - z_m)[\lambda_+ - (\lambda_+ - 2)z_m] + 36\lambda_+ \\ &\quad + 3L^2m^2[(1 + 3\epsilon^2)(\lambda_+ - 2)z_m^2 - 2(5 + 3\epsilon^2)(\lambda_+ - 1)z_m + 3(3 + \epsilon^2)\lambda_+]. \end{aligned} \quad (3.20)$$

Equation (3.18) and the Hawking temperature (2.4) show that (3.19) can be rewritten as

$$a^2 = \frac{3}{4(1-z_m)L^2} \left(\frac{T_c}{T}\right)^2 \left[1 - \left(\frac{T}{T_c}\right)^2\right], \quad (3.21)$$

where T_c is the critical temperature which is defined by

$$T_c = \frac{3}{8\pi L} \sqrt{\frac{\rho}{\tilde{b}}}. \quad (3.22)$$

According to the AdS/CFT dictionary, we obtain the relation

$$\langle O_+ \rangle \equiv LC_+ r_+^{\lambda_+} L^{-2\lambda_+} = LC_+ \left(\frac{8\pi T}{3}\right)^{\lambda_+}. \quad (3.23)$$

Thus, from Eqs. (3.17) and (3.21) we find that the expectation value $\langle O_+ \rangle$ is

$$\frac{\langle O_+ \rangle^{\frac{1}{\lambda_+}}}{T_c} = \Upsilon \frac{T}{T_c} \left\{ \left(\frac{T_c}{T}\right)^2 \left[1 - \left(\frac{T}{T_c}\right)^2\right] \right\}^{\frac{1}{2\lambda_+}}, \quad (3.24)$$

where Υ is a constant which is given by

$$\Upsilon = \frac{8\pi}{3} \left\{ \sqrt{\frac{3}{4(1-z_m)}} \frac{6 + 2m^2 L^2 (1-z_m)}{3z_m^{\lambda_+-1} [2z_m + \lambda_+(1-z_m)]} \right\}^{\frac{1}{\lambda_+}}. \quad (3.25)$$

In table I we present the critical temperature obtained analytically by fixing $z_m = 10/15$ and compare them with the results obtained by the numerical method. Selecting the appropriate matching point, we obtain consistent analytic result with that obtained numerically.

TABLE I: The critical temperature T_c obtained by the analytical method (left column) and the numerical method (right column). The matching point is set as $z_m = 10/15$. We have used $\rho = 1$ in the calculation.

ϵ	0.0	0.1	0.2	0.5	0.9
$m^2 L_{AdS}^2 = 0$	0.0492 0.0499	0.0504 0.0502	0.0515 0.0510	0.0544 0.0545	0.0577 0.0599
$m^2 L_{AdS}^2 = -1$	0.0566 0.0563	0.0573 0.0589	0.0580 0.0622	0.0602 0.0740	0.0631 0.0914
$m^2 L_{AdS}^2 = -2$	0.0779 0.0763	0.0749 0.0757	0.0734 0.0785	0.0725 0.0919	0.0741 0.1130

IV. ELECTRICAL CONDUCTIVITY IN HOŘAVA-LIFSHITZ BLACK-HOLE BACKGROUND

In the study of (2+1) and (3+1)-dimensional superconductors, Horowitz *et al.* [8] got a universal relation connecting the gap frequency in conductivity with the critical temperature T_c , which is

described by

$$\frac{\omega_g}{T_c} \approx 8, \quad (4.1)$$

with deviations of less than 8%. This is roughly twice the BCS value 3.5 indicating that the holographic superconductors are strongly coupled. However, the authors in Refs. [16, 42] found that this relation is not stable in the presence of the Gauss-Bonnet correction terms. And Cai *et al.* [36] got a relation

$$\frac{\omega_g}{T_c} \approx 13 \quad (4.2)$$

with the accuracy more than 93% for a planar Hořava-Lifshitz black hole with the condition of the detailed balance. We now examine this result for the Hořava-Lifshitz gravity considered here.

In order to compute the electrical conductivity, we should study the electromagnetic perturbation in this Hořava-Lifshitz black hole background, and then calculate the linear response to the perturbation. In the probe approximation, the effect of the perturbation of metric can be ignored. Assuming that the perturbation of the vector potential is translational symmetric and has a time dependence as $\delta A_x = A_x(r)e^{-i\omega t}$, we find that the Maxwell equation in the Hořava-Lifshitz black hole background reads

$$A_x'' + \frac{f'}{f}A_x' + \left(\frac{\omega^2}{f^2} - \frac{2\psi^2}{f}\right)A_x = 0, \quad (4.3)$$

where a prime denotes the derivative with respect to r . An ingoing wave boundary condition near the horizon is given by

$$A_x(r) \sim f(r)^{-\frac{2i\omega L^2}{3r_+}}. \quad (4.4)$$

In the asymptotic AdS region ($r \rightarrow \infty$), the general behavior should be

$$A_x = A^{(0)} + \frac{A^{(1)}}{r} + \dots. \quad (4.5)$$

By using AdS/CFT correspondence and the Ohm's law, we know that the conductivity can be expressed as [8]

$$\sigma = \frac{\langle J_x \rangle}{E_x} = -\frac{i\langle J_x \rangle}{\omega A_x} = \frac{A^{(1)}}{i\omega A^{(0)}}. \quad (4.6)$$

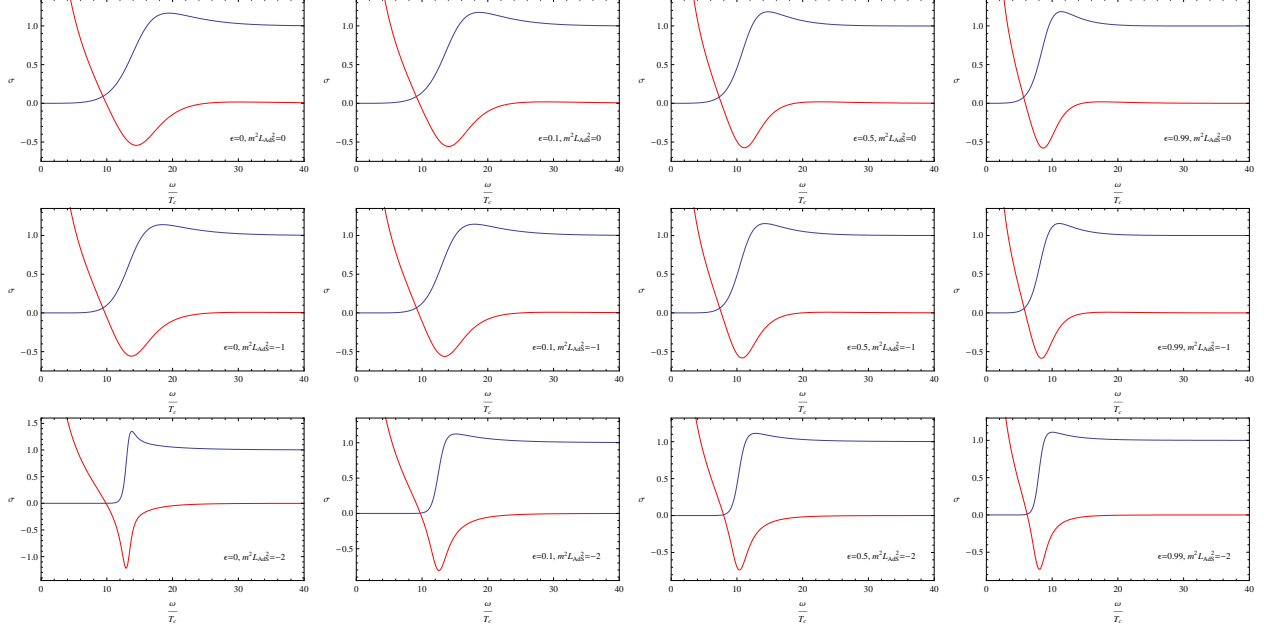


FIG. 4: (Color online) The conductivity of the superconductors for $\epsilon = 0, 0.1, 0.5$ and 0.99 with $m^2 L_{AdS}^2 = 0, -1$ and -2 . The solid (blue) line represents the real part of the conductivity, $Re(\sigma)$, and dashed (red) line is the imaginary part of the conductivity, $Im(\sigma)$.

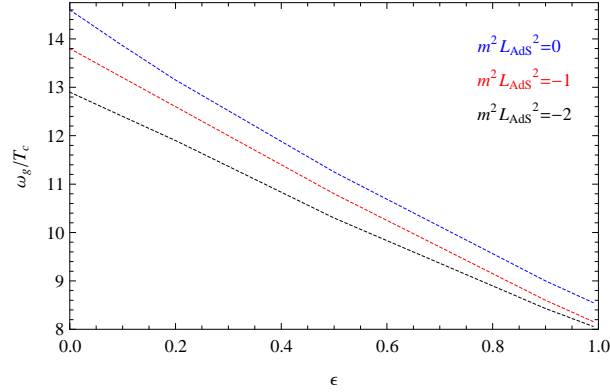


FIG. 5: (Color online) The ratio ω_g/T_c as a function of the balance parameter with fixed values $m^2 L_{AdS}^2$.

In Fig. 4 we plot the frequency dependent conductivity obtained by solving the Maxwell equation numerically for $\epsilon = 0, 0.1, 0.5$ and 0.99 with $m^2 L_{AdS}^2 = 0, -1$ and -2 . We find a gap in the conductivity with the gap frequency ω_g . For the same value of $m^2 L_{AdS}^2$, the gap frequency ω_g decreases with the increase of the constant ϵ . In each plot, the real part of the conductivity, $Re[\sigma]$, approaches to a limit when the frequency grows. The limit for the case $\epsilon = 0$ is one, but general it increases as ϵ increases. The imaginary part of conductivity $Im[\sigma]$ becomes zero when $\omega \rightarrow \infty$, but it goes to

TABLE II: The ratio ω_g/T_c for different values of the constant ϵ with $m^2 L_{AdS}^2 = 0, -1$ and -2 .

ϵ	0	0.1	0.2	0.5	0.9	0.99
$m^2 L_{AdS}^2 = 0$	14.6	13.9	13.2	11.3	9.0	8.6
$m^2 L_{AdS}^2 = -1$	13.8	13.2	12.6	10.8	8.6	8.2
$m^2 L_{AdS}^2 = -2$	12.9	12.4	11.9	10.3	8.4	8.1

infinity when the frequency approaches zero.

In Fig. 5 we present the ratio ω_g/T_c as a function of the balance parameter with fixed values $m^2 L_{AdS}^2 = 0, -1$ and -2 , which shows that the ratio ω_g/T_c almost linearly decreases with the increase of the balance constant.

From Figs. 4, 5 and table II, we find that the ratio of the gap frequency in conductivity ω_g to the critical temperature T_c in this black hole reduces to Cai's result $\omega_g/T_c \approx 13$ [36] found in the Hořava-Lifshitz black hole with the condition of the detailed balance for small ϵ , while it tends to the Horowitz-Roberts relation $\omega_g/T_c \approx 8$ as $\epsilon \rightarrow 1$. Our result provides a bridge between the two kinds of relations.

V. CONCLUSIONS

The behavior of the holographic superconductors in the Hořava-Lifshitz gravity has been investigated in this manuscript by introducing a complex scalar field and a Maxwell field in a planar black-hole background. We first present a detailed analysis of the condensation of the operator \mathcal{O}_+ by the numerical and analytical methods for the Hořava-Lifshitz black hole without the condition of the detailed balance. We obtain consistent analytical results with those obtained numerically by selecting the appropriate matching point. It is found that, as the parameter of the detailed balance ϵ increases with fixed mass of the scalar field m , the condensation gap becomes smaller, corresponding to the larger critical temperature, which means that the scalar hair can be formed easier for the larger ϵ . And it is shown that, for the same ϵ , the condensation gap becomes larger if m^2 becomes less negative, which means that it is harder for the scalar hair to form as the mass of the scalar field becomes larger. We then studied the electrical conductivity in the Hořava-Lifshitz black-hole background and find that the ratio of the gap frequency in conductivity to the critical temperature, ω_g/T_c , almost linearly decreases with the increase of the balance constant. For $\epsilon = 0$ the ratio reduces to Cai's result $\omega_g/T_c \approx 13$ [36] found in a Hořava-Lifshitz black hole with the condition of the detailed balance, while as $\epsilon \rightarrow 1$ it

tends to the Horowitz-Roberts relation $\omega_g/T_c \approx 8$ [8] obtained in the AdS Schwarzschild black hole. It is interesting to note that our result provides a bridge between the results for the Hřrava-Lifshitz theory with the condition of the detailed balance and Einstein’s gravity.

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